COT 3100 In-class Exercise 10

Name: USF ID:

**Problem 1: Second-Order Linear Homogeneous Recurrence Relations with Constant Coefficients – Distinct Roots.**

Suppose a sequence satisfies the below given recurrence relation and initial conditions. Find an explicit formula for the sequence.

for all integers

Answer:

The characteristic equation is So, , hence, distinct roots and.

Follow the distinct roots theorem that for some constants *C* and *D, , , , . . .* satisfies the equation

for all integers

Since , then

Solving the above equations, obtain .

Hence, for all integers

**Problem 2: Second-Order Linear Homogeneous Recurrence Relations with Constant Coefficients – Single Root.**

Suppose a sequence satisfies the below given recurrence relation and initial conditions. Find an explicit formula for the sequence.

for all integers

Answer:

The characteristic equation is So,, hence, single root.

Follow the single roots theorem that for some constants *C* and *D, , , , . . .* satisfies the equation

for all integers

Since , then

Solving the above equations, obtain .

Hence, for all integers

**Problem 3: Loop Invariants**

*[pre-condition: and i]*

**while** ()

1. i := i + 1

**end while**

[post condition: ]

Loop invariant: is “.”

**Proof:**

**I. Basis Property:** *I (*0*)* is “*i* = 1 and *sum* = *A*[1].” According to the pre-condition, this statement is true.

**II. Inductive Property:** Suppose *k* is a nonnegative integer such that *G* ∧ *I (k)* is true before an iteration of the loop. Means: , and . After execution of statement 1, . And after execution of statement 2,. Thus after the loop iteration, *I (k* + 1*)* is true.

**III. Eventual Falsity of Guard:** The guard *G* is the condition . By I and II, it is known that for all integers *n* ≥ 1, after *n* iterations of the loop, *I (n)* is true. Hence, after *m* − 1 iterations of the loop, *I (m)* is true, which implies that *i* = *m* and *G* is false.

**IV. Correctness of the Post-Condition:** Suppose that *N* is the least number of iterations after which *G* is false and *I (N)* is true. Then (since *G* is false) *i* = *m* and (since *I (N)* is true) *i* = *N* +1 and *sum* =

*A*[1] + *A*[2]+· · ·+ *A*[*N* + 1]. Putting these together gives *m* = *N* + 1, and so *sum* = *A*[1] + *A*[2]+· · ·+

*A*[*m*], which is the post-condition.